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SUMMARY

An analysis is presented for fully developed heat transfer for laminar flow of a power-law non-Newtonian fluid in a circular tube with arbitrary circumferential wall temperature or heat flux. The results allow calculation of temperature or wall heat-flux variations for various velocity profiles when the tube has a uniform axial heat input and an arbitrary peripheral wall-temperature or heat-flux variation. The circumferential variation of Nusselt modulus with respect to these temperature or heat-flux variations and velocity profiles is shown. It is found that heat-transfer analyses based on the assumption of a parabolic velocity distribution will introduce serious errors when the fluid exhibits a nonlinear shearing-stress - shearing-rate relation, so that the actual behavior of the velocity profile must be considered.

INTRODUCTION

The study of heat-transfer problems for fully developed laminar or turbulent flow in a circular tube with longitudinally uniform or varying wall temperature or heat flux has been pursued by analytical methods over a long period of time. The situation of circumferentially varying wall temperature or heat flux, of special interest in nuclear reactor and heat-exchanger applications, has, on the other hand, been studied only recently. The case of heat transfer to laminar Poiseuille flow in a tube with arbitrary circumferential heat flux has been considered in reference 1. The problem of turbulent heat transfer in a tube with variable circumferential heat flux has been treated in reference 2. Heat transfer to turbulent flow in a tube with circumferentially varying temperature has been considered in reference 3.

The desire to make heat-exchange equipment compact and to increase the amount of heat transferred per unit of pumping power frequently lead the design engineer to consider passage sizes such that the equipment operates well into the laminar-flow region. Laminar-flow considerations, therefore, remain of considerable practical interest. Many problems that occur in physical situations, moreover, involve a velocity distribution other than that of a parabolic shape (Poiseuille flow). A few examples are the flow of nuclear fuel slurries, the flow of liquid metals, and the flow of confined plasmas. It is desirable, therefore, to extend the analyses of references 1 to 3 to include other possible laminar velocity-distribution profiles.

The present investigation is concerned with the fully developed laminar power-law non-Newtonian flow in a round tube with constant axial heat input but variable circumferential wall temperature or heat flux. The Newtonian fluid is a fluid for which the diagram relating shear stress and rate of shear, the so-called flow curve, is a straight line of slope μ, where μ is the Newtonian viscosity. The single constant μ completely characterizes the Newtonian fluid.

Non-Newtonian fluids are those for which the flow curve is not linear, that is, the viscosity is not constant but depends on a factor such as the rate of shear in the fluid. The logarithmic plot of shear stress and rate of shear for these fluids is often found to be linear. As a result, a functional relation defined as the power law is used to characterize fluids of this type.

The results of this investigation should provide some insight into the influence of the velocity distribution on the heat-transfer results. Since the convective term in the energy equation involves the velocity distribution, the first step in the analysis is to specify the velocity variation over the tube cross section. The power-law model, which has found widespread use as an approximate representation of pseudoplastic rheology, is considered in the present analysis. The velocity profile of steady non-Newtonian power-law fluids in circular tubes has been treated in reference 4, and the results are used in the present investigation. With the velocity distribution specified, the energy equation can be considered. The study is divided into two parts.

In the first part of the present work, the heat transfer to fully developed tube flow with arbitrary circumferential wall temperature is considered following the method of Sparrow and Lin (ref. 3). In the second part, the heat transfer to fully developed tube flow with arbitrary circumferential wall heat flux is considered by utilizing the method of analysis of Reynolds (ref. 1). Throughout the analyses, the flow is assumed thermally fully developed, the heat flux is taken as uniform in the axial direction, viscous dissipation and axial conduction are neglected, and fluid properties are assumed constant. These idealizations are familiar ones and require no discussion.

VARIABLE CIRCUMFERENTIAL WALL TEMPERATURE

There is considered a tube flow along which there is a uniform heat-transfer rate per unit length Q. (Symbols are defined in the appendix.) The tube wall temperature t_w is allowed to vary around the circumference in an arbitrary manner as given by the Fourier expansion (ref. 3)

$$\frac{t_w(\varphi) - \bar{t}_w}{\frac{\bar{q}r_o}{k}} = \sum_{n=1}^{\infty} a_n \cos n\varphi + b_n \sin n\varphi \tag{1}$$

in which $\bar{q}$ is the average heat-flux rate per unit heated area ($\bar{q} = Q/2\pi r_o$) and $\bar{t}_w$ is the circumferentially averaged wall temperature at a given axial position.

The starting point of the analysis is the energy equation for convective heat transfer with fully established velocity and temperature profiles

$$\frac{u}{\alpha}\frac{\partial t}{\partial x} = \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2} \tag{2}$$

The fully developed heat-transfer condition for a uniform wall heat flux independent of x is characterized by the fact that the temperature profile is similar in shape for all values of x, and the temperature values rise in a linear fashion in the longitudinal direction. Therefore, the temperature gradient $\partial t/\partial x$, a constant, is given by

$$\frac{\partial t}{\partial x} = \frac{2\bar{q}}{\rho c_p \bar{u} r_o} \tag{3}$$

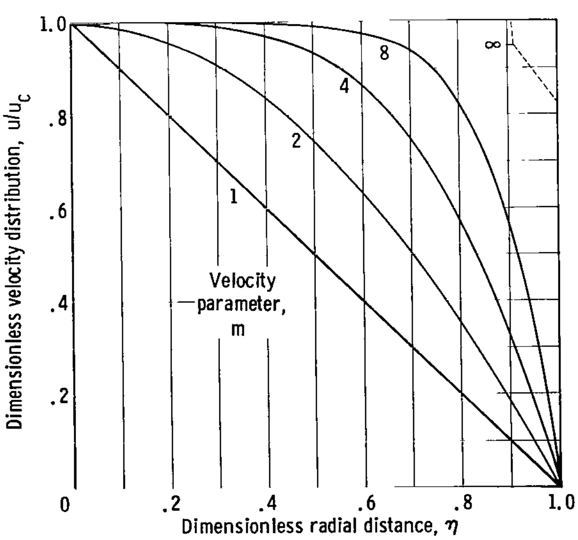


Figure 1. - Velocity distribution in circular tube.

where $\bar{q}$ is taken as positive from wall to fluid.

The velocity distribution is given by (ref. 4)

$$u = \bar{u}\,\frac{m+2}{m}\,(1 - \eta^m) = u_c\,(1 - \eta^m) \tag{4}$$

where m is a constant for a particular fluid; m is a measure of the degree of non-Newtonian behavior, and the greater the departure from 2 the more pronounced are the non-Newtonian properties. For $m \geq 2$, the fluid is termed a pseudoplastic fluid. The power-law equation is also often applicable for dilatant fluids (ref. 4), but in this case the velocity parameter m is less than 2 ($1 \leq m \leq 2$). The Poiseuille flow is represented by $m = 2$, while $m \to \infty$ yields slug flow. Distributions as a function of m are shown in figure 1.

Equation (2), written in terms of dimensionless variables, then becomes

$$\frac{\partial^2 \tau}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \tau}{\partial \eta} + \frac{1}{\eta^2} \frac{\partial^2 \tau}{\partial \varphi^2} = \frac{2(m + 2)}{m} (1 - \eta^m) \tag{5}$$

This equation is to be solved subject to appropriate boundary conditions. The solution of equation (5) for τ can be written as (ref. 3)

$$\tau(\eta, \varphi) = \tau_d(\eta) + \sum_{n=1}^{\infty} H_n(\eta) (a_n \cos n\varphi + b_n \sin n\varphi) \tag{6}$$

The functions $\tau_d(\eta)$ and $H_n(\eta)$ are determined by substituting equation (6) into equation (5) and then evaluating the result such that

$$\frac{d^2 \tau_d}{d\eta^2} + \frac{1}{\eta} \frac{d\tau_d}{d\eta} = \frac{2(m + 2)}{m} (1 - \eta^m) \tag{7}$$

$$\frac{d^2 H_n}{d\eta^2} + \frac{1}{\eta} \frac{dH_n}{d\eta} - \frac{n^2}{\eta^2} H_n = 0 \tag{8}$$

The function τ_d, which is the fully developed temperature solution for axisymmetric heating, is the solution to equation (7) subject to the boundary conditions

$$\frac{d\tau_d}{d\eta} = 0 \quad \text{at} \quad \eta = 0 \qquad \text{(symmetry)}$$

$$\tau_d = 0 \quad \text{at} \quad \eta = 1 \qquad \text{(specified wall temperature distribution, eq. (1))}$$

The function $H_n(\eta)$ is the solution to equation (8) subject to the boundary conditions

$$H_n(0) = 0 \qquad \text{(angular symmetry)}$$

$$H_n(1) = 1 \qquad \text{(specified wall temperature distribution, eq. (1))}$$

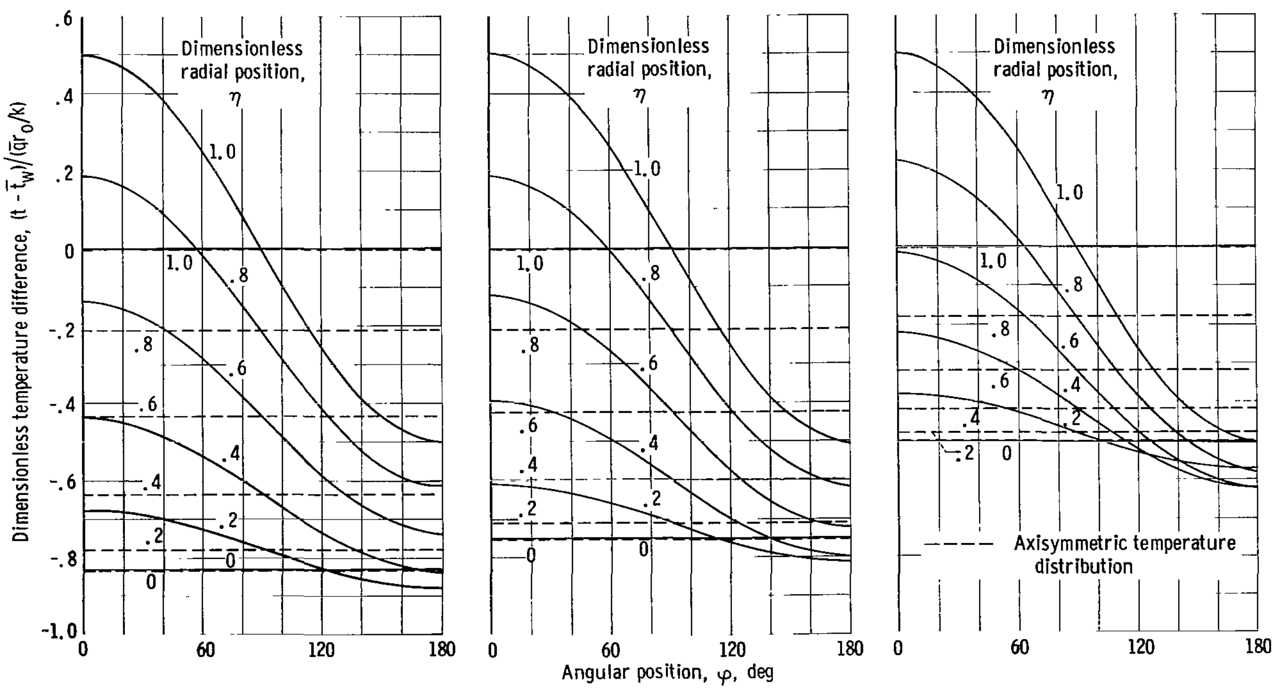
The functions $\tau_d(\eta)$ and $H_n(\eta)$ are readily found to be, respectively,

$$\tau_d = \frac{2(m+2)}{m}\left[\frac{\eta^2}{4} - \frac{\eta^{m+2}}{(m+2)^2} - \frac{1}{4} + \frac{1}{(m+2)^2}\right] \tag{9}$$

$$H_n(\eta) = \eta^n \tag{10}$$

Now that τ_d and H_n are known, they can be substituted into equation (6) to obtain the complete solution for the temperature distribution in the tube, which is

$$\tau(\eta, \varphi) = \frac{m+2}{2m}\eta^2 - \frac{2}{m(m+2)}\eta^{m+2} - \frac{(m+2)^2 - 4}{2m(m+2)} + \sum_{n=1}^{\infty} \eta^n (a_n \cos n\varphi + b_n \sin n\varphi) \tag{11}$$



(a) Velocity parameter, 1.

(b) Velocity parameter, 2.

(c) Velocity parameter, ∞.

Figure 2. - Illustrative circumferential variations of fluid temperatures corresponding to wall-temperature variation $(t_w - \bar{t}_w)/(\bar{q}r_0/k) = 0.5 \cos \varphi$.

To illustrate the results, temperature profiles are plotted as a function of angular position φ at various radial locations η in figure 2 for an illustrative case where $a_1 = 0.5$, $a_n = 0$ for $n = 2, 3, \ldots$, and $b_n = 0$ for $n = 1, 2, \ldots$. The values of m represented are $m = 1, 2$, and ∞. In figure 2(a) $m = 1$ represents a linear velocity profile; $m = 2$ corresponds to Poiseuille flow (fig. 2(b)); and $m \rightarrow \infty$ represents slug flow (fig. 2(c)). The angular variations of temperature, at a given radial location, are very nearly the same for the three values of m represented. At a given angular position, however, the radial variation of temperature is greatest for $m = 1$ and least for $m \rightarrow \infty$. At the tube centerline $\eta = 0$, the temperature distribution, as expected, is independent of angular location. Also shown in figure 2 are curves for a uniform, or axisymmetric, wall-temperature distribution. As noted in the previous example, the radial variations of temperature are greatest for $m = 1$ and least for $m \rightarrow \infty$.

In order to determine the local heat-transfer coefficient, or Nusselt number, it is necessary to know the fluid bulk temperature t_b and the local wall heat flux $q(\varphi)$. The fluid bulk temperature is obtained as follows:

$$t_b = \frac{1}{\pi r_o^2 \bar{u}} \int_0^{2\pi} \int_0^{r_o} utr\, dr\, d\varphi \tag{12}$$

Equation (12) may be written in dimensionless form as

$$\tau_b = \frac{1}{\pi} \int_0^{2\pi} \int_0^{1} \left(\frac{u}{\bar{u}}\right) \tau\eta\, d\eta\, d\varphi \tag{13}$$

When equation (11), in addition to equation (4), is used to evaluate the integrals of equation (13), the cosine and sine terms all drop out in the integration over φ. The remaining terms may be readily integrated to yield

$$\tau_b = \frac{(m + 2)^3 - 8}{4m(m + 2)(m + 4)} - \frac{(m + 2)^2 - 4}{2m(m + 2)} \tag{14a}$$

It is convenient to define the right side of equation (14a) as $-2f(m)$, that is, as a function of the parameter m, so that

$$\tau_b = -2f(m) \tag{14b}$$

The wall- to bulk-temperature difference $t_w(\varphi) - t_b$ is determined by combining equations (1) and (14) to yield

$$\frac{t_w(\varphi) - t_b}{\frac{\bar{q}r_o}{k}} = \frac{\left[t_w(\varphi) - \bar{t}_w\right] - (t_b - \bar{t}_w)}{\frac{\bar{q}r_o}{k}}$$

$$= 2f(m) + \sum_{n=1}^{\infty} a_n \cos n\varphi + b_n \sin n\varphi \tag{15}$$

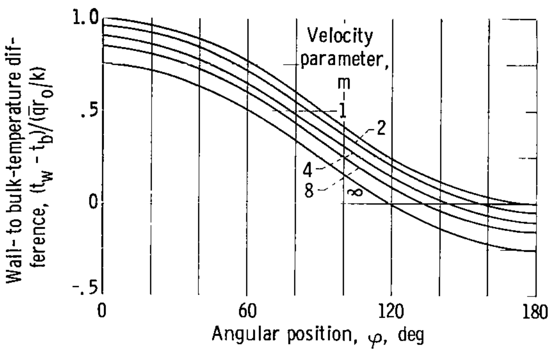


Figure 3. - Variation of wall- to bulk-temperature difference for prescribed wall-temperature variation $(t_w - \bar{t}_w)/(\bar{q}r_o/k) = 0.5 \cos \varphi$.

Equation (15) has been plotted in figure 3 as a function of the parameters φ and m for the previously mentioned illustrative case, namely, $\left[t_w(\varphi) - \bar{t}_w\right]/(\bar{q}r_o/k) = 0.5 \cos \varphi$. Note that the wall- to bulk-temperature difference varies substantially around the periphery of the tube. The wall- to bulk-temperature difference becomes zero when the bulk temperature of the fluid is equal to the wall temperature. Since the Nusselt number is inversely proportional to the wall- to bulk-temperature difference, infinities in the Nusselt number will occur when the differences are zero.

The local wall heat flux can be evaluated from Fourier's law

$$q(\varphi) = k\left(\frac{\partial t}{\partial r}\right)_{r=r_o} \tag{16a}$$

or, in terms of dimensionless variables τ and η,

$$q(\varphi) = \bar{q}\left(\frac{\partial \tau}{\partial \eta}\right)_{\eta=1} \tag{16b}$$

Applying this to equation (11) gives the result for the variation of wall heat flux with φ as

$$\frac{q}{\bar{q}} = 1 + \sum_{n=1}^{\infty} n(a_n \cos n\varphi + b_n \sin n\varphi) \tag{17}$$

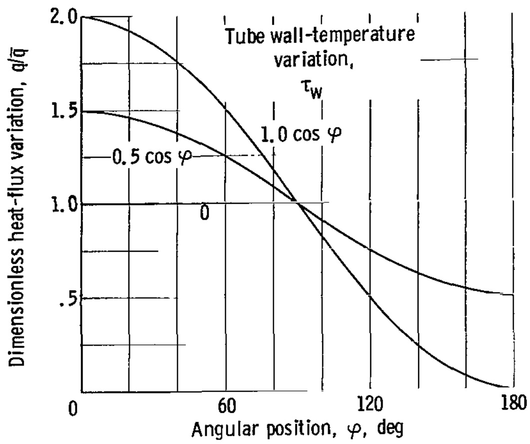


Figure 4. - Illustrative heat-flux variation around tube periphery for given circumferential wall-temperature variation.

It is interesting to note that $q/\bar{q}$ is independent of the velocity field.

Figure 4 illustrates the wall heat-flux variation around the tube circumference, as given by equation (17), for tube wall-temperature variations $\left[t_w(\varphi) - \bar{t}_w\right]/(\bar{q}r_o/k) = 0$, 0.5 cos φ, and 1.0 cos φ.

Now that the wall- to bulk-temperature difference $t_w - t_b$ and the wall heat flux q are known, the variation of the heat-transfer coefficient, or the Nusselt number, can be determined.

A circumferentially averaged Nusselt number $\overline{Nu}$ may be defined as

$$\overline{Nu} \equiv \frac{2r_o \int_0^{2\pi} q(\varphi)d\varphi}{2\pi k(\bar{t}_w - t_b)} = \frac{2\bar{q}r_o}{k(\bar{t}_w - t_b)} \tag{18}$$

When equation (14) is used, the average Nusselt number is determined as

$$\overline{Nu} = \frac{1}{f(m)} \tag{19}$$

A circumferentially varying Nusselt number $Nu(\varphi)$ may be defined as

$$Nu(\varphi) \equiv \frac{2r_o q(\varphi)}{k\left[t_w(\varphi) - t_b\right]} \tag{20}$$

Substituting $q(\varphi)$ from equation (17) and $t_w(\varphi) - t_b$ from equation (15) gives the Nusselt modulus $Nu(\varphi)$ as

$$Nu(\varphi) = \frac{1 + \sum_{n=1}^{\infty} n(a_n \cos n\varphi + b_n \sin n\varphi)}{f(m) + \frac{1}{2}\sum_{n=1}^{\infty} a_n \cos n\varphi + b_n \sin n\varphi} \tag{21}$$

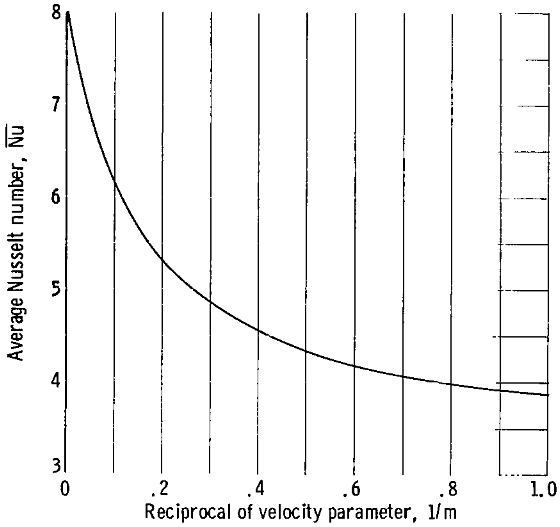


Figure 5. - Average Nusselt number for laminar flow in circular tube.

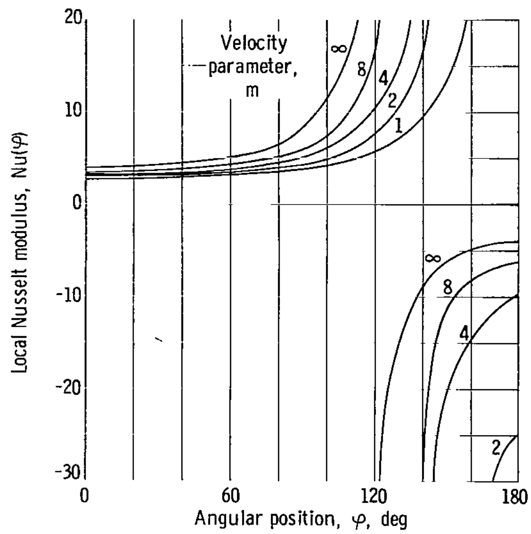


Figure 6. - Local Nusselt modulus variation for prescribed wall temperature $(t_w - \bar{t}_w)/(\bar{q}r_0/k) = 0.5 \cos \varphi$.

It should be noted for $a_n = b_n = 0$, or constant circumferential wall temperature, the Nusselt modulus reduces to $\overline{Nu}$.

The average Nusselt number $\overline{Nu}$ (eq. (19)), plotted in figure 5 as a function of $1/m$, corresponds to the axisymmetric heat-transfer situation, and hence the average heat-transfer performance is equal to the performance for uniform peripheral heat transfer. By plotting the average Nusselt number $\overline{Nu}$ as a function of the reciprocal of the velocity parameter, the physically interesting velocity profiles are represented by the abscissa variation $0 \leq 1/m \leq 1.0$. The average Nusselt number is greatest for slug flow $(m \rightarrow \infty)$ and least for a linear velocity profile $(m = 1)$.

The local Nusselt number (eq. (21)) has been evaluated as a function of the parameters φ and m for $\left[t_w(\varphi) - \bar{t}_w\right]/(\bar{q}r_0/k) = 0.5 \cos \varphi$ and is plotted in figure 6. The local Nusselt number, for a given value of m, varies significantly around the tube from the uniform heat-flux value shown in figure 5. The Nusselt number becomes infinite at the point where the wall temperature equals the fluid bulk temperature and becomes negative when the wall temperature is less than the bulk temperature. An appreciable variation of the heat-transfer coefficient around the tube periphery, therefore, occurs for a given value of m, and, hence, it is unsatisfactory to use the average Nusselt number $\overline{Nu}$, or even the local heat flux, together with the heat-transfer coefficient obtained from the uniform heat-input solution, to determine the wall-temperature variation around the tube periphery.

Another factor that can influence the Nusselt number, or heat-transfer coefficient, is the shape of the velocity profile. Heat-transfer analysis based, therefore, on the assumption of a parabolic velocity distribution will introduce serious errors when the

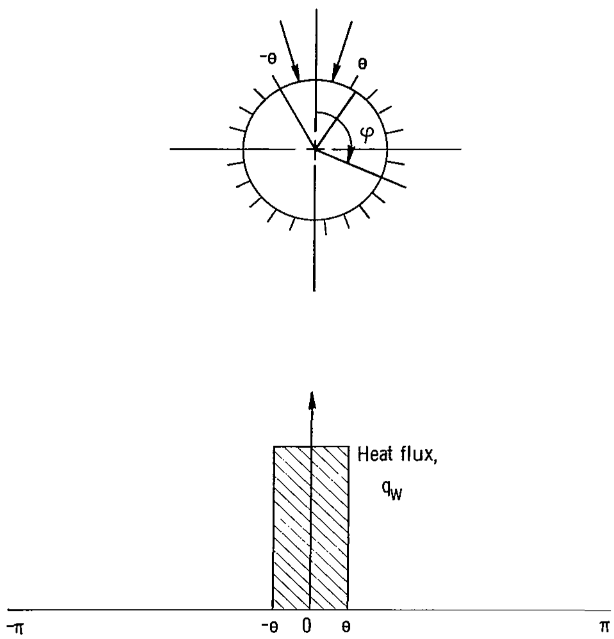


Figure 7. - Tube with uniform heat flux q_w over $-\theta \leq \varphi \leq \theta$. Remainder of boundary insulated.

fluid has pronounced non-Newtonian behavior, and the actual behavior of the velocity profile must be considered in a heat-transfer analysis.

The significant variations in the Nusselt number exhibited in figure 6 open to question the utility of a heat-transfer coefficient under conditions of a variable surface temperature. Independent consideration of equations (15) and (17) may prove more useful for a quantitative heat-transfer evaluation in this situation. In any event, it is clear that both the wall-temperature variation and the fluid-velocity distribution markedly alter the heat-transfer conditions around the tube periphery.

VARIABLE CIRCUMFERENTIAL WALL HEAT FLUX

The method of analysis follows the approach taken by reference 1 and is as follows: First, a solution is obtained for the case of a tube with constant heat flux over a portion of its circumference, insulated over the remainder (fig. 7). Then the result is generalized to the situation of arbitrary circumferential variations of wall heat flux.

The starting point for the variable circumferential wall heat-flux problem is also the energy conservation equation (eq. (2)), but now an energy balance in the fully developed thermal situation produces the result that

$$\frac{\partial t}{\partial x} = \frac{2q_w}{\rho c p \bar{u} r_o} \frac{\theta}{\pi} \tag{22}$$

Equation (2), written in terms of dimensionless variables, becomes

$$\frac{\partial^2 T}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial T}{\partial \eta} + \frac{1}{\eta^2}\frac{\partial^2 T}{\partial \varphi^2} = a(1 - \eta^m) \tag{23}$$

Equation (23) is to be solved subject to the boundary conditions

$$k\left(\frac{\partial t}{\partial r}\right)_{r=r_o} = q \equiv q_w \qquad -\theta \leq \varphi \leq \theta \tag{24a}$$

$$k\left(\frac{\partial t}{\partial r}\right)_{r=r_o} = 0 \qquad \theta < \varphi < 2\pi - \theta \tag{24b}$$

where q_w is a constant. Written in dimensionless variables,

$$\left(\frac{\partial T}{\partial \eta}\right)_{\eta=1} = \frac{q_w r_o}{k} \qquad -\theta \leq \varphi \leq \theta \tag{24c}$$

$$\left(\frac{\partial T}{\partial \eta}\right)_{\eta=1} = 0 \qquad \theta < \varphi < 2\pi - \theta \tag{24d}$$

The analysis is simplified in the following way. Assume a solution to equation (23) of the form

$$T(\eta, \varphi) = F(\eta, \varphi) + G(\eta) \tag{25}$$

The functions F and G are determined by substituting equation (25) into equation (23) and evaluating the result such that

$$\frac{\partial^2 F}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial F}{\partial \eta} + \frac{1}{\eta^2}\frac{\partial^2 F}{\partial \varphi^2} = 0 \tag{26}$$

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta \frac{dG}{d\eta}\right) = a(1 - \eta^m) \tag{27}$$

The function $G(\eta)$ is considered first. Equation (27) can be integrated directly; the resulting expression for $G(\eta)$ is

$$G(\eta) = a\left[\frac{\eta^2}{4} - \frac{\eta^{m+2}}{(m+2)^2}\right] + B_o \tag{28}$$

where the boundary condition $dG/d\eta = 0$ at $\eta = 0$ has been used and B_o is a constant of integration.

The function $F(\eta, \varphi)$, which is the solution to Laplace's equation (eq. (26)), has been shown to be of the form (ref. 1)

$$F = c_o + \sum_{n=1}^{\infty} c_n \eta^n \cos n\varphi \tag{29}$$

Combining equations (28) and (29) in accordance with equation (25) yields

$$T = a\left[\frac{\eta^2}{4} - \frac{\eta^{m+2}}{(m+2)^2}\right] + D_o + \sum_{n=1}^{\infty} c_n \eta^n \cos n\varphi \tag{30}$$

where the coefficients c_n and the constant $D_o \equiv c_o + B_o$ remain to be determined.

The series expansion coefficients c_n are evaluated to satisfy the boundary conditions, equations (24c) and (24d). Differentiating equation (30) with respect to η and substituting the result in equations (24c) and (24d) yield

$$\left.\frac{\partial F}{\partial \eta}\right|_{\eta=1} = \sum_{n=1}^{\infty} nc_n \cos n\varphi = \frac{q_w r_o}{k}\left(1 - \frac{\theta}{\pi}\right) \qquad -\theta \leq \varphi \leq \theta \tag{31a}$$

$$\sum_{n=1}^{\infty} nc_n \cos n\varphi = -\frac{q_w r_o}{k}\,\frac{\theta}{\pi} \qquad \theta < \varphi < 2\pi - \theta \tag{31b}$$

Then, by a Fourier analysis

$$c_n = \left(\frac{2}{\pi n^2}\right)\left(\frac{q_w r_o}{k}\right) \sin n\theta \tag{32}$$

and equation (30) takes the form

$$T = a\left[\frac{1}{4}\eta^2 - \frac{\eta^{m+2}}{(m+2)^2}\right] + D_o + \sum_{n=1}^{\infty} \frac{2q_w r_o}{n^2 \pi k} \eta^n \sin n\theta \cos n\varphi \tag{33}$$

The remaining unknown, the constant D_o, is evaluated such that the mixed mean temperature t_b is given by equation (12). In terms of the temperature difference T, equation (12) may be written as

$$\int_0^{2\pi} \int_0^1 (1 - \eta^m) T\eta \, d\eta \, d\varphi = 0 \tag{34}$$

Using equation (33) in equation (34) produces the result for D_o as

$$D_o = -\frac{q_w r_o}{k} \frac{\theta}{\pi} \left[\frac{(m+2)^2}{4m^2} - \frac{(m+2)^2 + 4}{m^2(m+4)} + \frac{2}{m^2(m+2)} \right] \tag{35}$$

Combining equations (33) and (35) yields the complete distribution for the temperature difference T as

$$T = \frac{q_w r_o}{k} \left\{ \frac{\theta}{\pi} \left[\frac{m+2}{2m} \eta^2 - \frac{2}{m(m+2)} \eta^{m+2} - \frac{(m+2)^2}{4m^2} \right. \right.$$

$$\left. \left. + \frac{(m+2)^2 + 4}{m^2(m+4)} - \frac{2}{m^2(m+2)} \right] + \sum_{n=1}^{\infty} \frac{2}{\pi n^2} \eta^n \sin n\theta \cos n\varphi \right\} \tag{36}$$

When the wall heat flux is specified, the wall temperature is the unknown quantity that is usually of most practical interest. The wall-temperature difference T_w can be found by evaluating equation (36) at $\eta = 1$ with the result

$$\frac{T_w}{\frac{2q_w r_o}{k}} = \frac{\theta}{\pi} \left[\frac{8 - (m+2)^3}{8m(m+2)(m+4)} + \frac{(m+2)^2 - 4}{4m(m+2)} \right] + \sum_{n=1}^{\infty} \frac{1}{n^2 \pi} \sin n\theta \cos n\varphi \tag{37}$$

The term in brackets is equal to $f(m)$, and, hence, equation (37) can be rewritten in a more convenient form as

$$\frac{T_w}{\frac{2q_w r_o}{k}} = \frac{\theta}{\pi} f(m) + \sum_{n=1}^{\infty} \frac{1}{n^2 \pi} \sin n\theta \cos n\varphi \tag{38}$$

Equation (38) is the inverse of the local Nusselt number:

$$Nu(\varphi) = \frac{2q_w r_o}{kT_w} = \frac{1}{\frac{\theta}{\pi} f(m) + \sum_{n=1}^{\infty} \frac{1}{n^2 \pi} \sin n\theta \cos n\varphi} \tag{39}$$

This is the Nusselt number variation around the heated portion of the tube with a uniform heat input over $-\theta \leq \varphi \leq \theta$ and insulated over $\theta < \varphi < 2\pi - \theta$. The influence of the velocity distribution is felt through the function f(m).

The foregoing results may be extended to include the arbitrary circumferential wall heat flux $q_w(\varphi)$ by the method presented in reference 1. Since the energy equation is linear, a superposition principle can be used to generalize the results obtained for the case of a tube with uniform heat flux over a portion of its circumference and insulated over the remainder to the case where the circumferential heat flux is arbitrary. Little is to be gained by a repetition of the procedure, and, hence, the reader is referred to reference 1 for details. The response in wall-temperature difference for an arbitrary circumferential heat flux is found as

$$T_w(\varphi) = \int_0^{2\pi} \frac{2r_o q_w(\xi)}{k} \left[f(m) + \sum_{n=1}^{\infty} \frac{1}{n} \cos n(\varphi - \xi) \right] \frac{d\xi}{2\pi} \tag{40}$$

Equation (40) represents the general solution to the arbitrary circumferential flux and non-Newtonian velocity-distribution problem.

For the special case of uniform peripheral heat flux, $q_w(\varphi) = \bar{q}$ = constant, and equation (40) yields

$$T_w = \frac{2\bar{q} r_o}{k} f(m) \tag{41}$$

and, hence, as expected, the local Nusselt number is given by

$$Nu = \frac{2\bar{q} r_o}{kT_w} = \frac{1}{f(m)} \tag{42}$$

It should be noted that equation (42) is identical to equation (19). This is to be expected, since, as pointed out earlier, equation (19) corresponds to uniform peripheral heat transfer, as does equation (42).

There is now considered as an illustrative case a cosine circumferential heat-flux distribution of the form

$$q_w(\varphi) = \bar{q}(1 + b \cos \varphi) \tag{43}$$

Such a flux distribution might be representative of that found in a nuclear reactor. Reference 1 has examined how this circumferential heat-flux variation, for various values of b, influences the wall temperatures for Poiseuille flow in a tube. The purpose here in considering equation (43) is to determine the combined influence of circumferential heat-flux variation (in particular, that represented by eq. (43)) and velocity distribution on the convection process. From equation (40), the wall-temperature distribution for this heat-flux distribution is given by

$$T_w = \frac{2\bar{q}r_o}{k} \int_0^{2\pi} (1 + b \cos \xi)\left[f(m) + \sum_{n=1}^{\infty} \frac{\cos n(\varphi - \xi)}{n}\right]\frac{d\xi}{2\pi} \tag{44}$$

Carrying out the integration gives the result

$$T_w(\varphi) = \frac{2\bar{q}r_o}{k}\left[f(m) + \frac{b}{2} \cos \varphi\right] \tag{45}$$

The average wall-temperature difference $\overline{T}_w$ is obtained by integration of equation (45) around the tube periphery:

$$\overline{T}_w = \frac{1}{2\pi} \int_0^{2\pi} T_w(\varphi)d\varphi = \left(\frac{2\bar{q}r_o}{k}\right) f(m) \tag{46}$$

Dividing equation (45) by equation (46) yields the ratio of the local wall-temperature difference to the average wall-temperature difference as

$$\frac{T_w(\varphi)}{\overline{T}_w} = 1 + \frac{b}{2f(m)} \cos \varphi \tag{47}$$

The appearance of both the parameter b and the function f(m) in equation (47) indicates

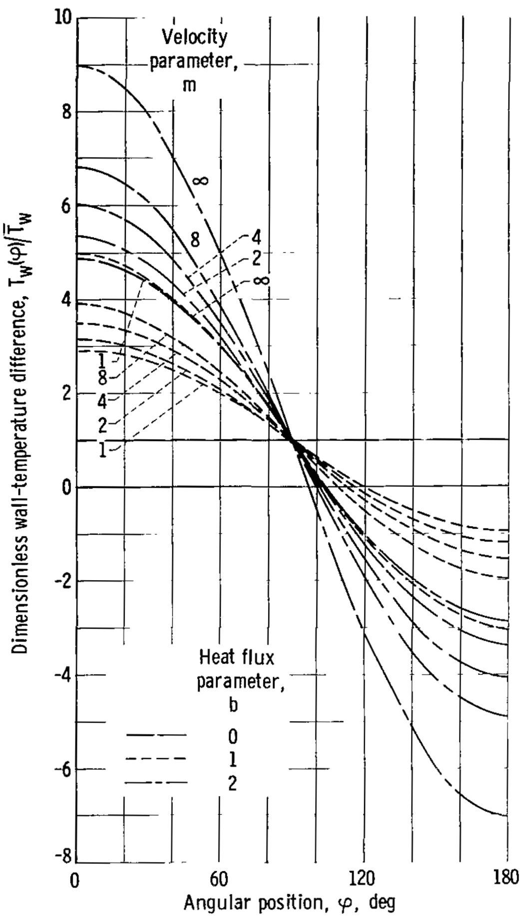


Figure 8. - Ratio of local wall-temperature difference to average wall-temperature difference for prescribed heat flux $q(\varphi)/\overline{q} = 1 + b \cos \varphi$.

that the heat-flux parameter and the velocity distribution influence the ratio $T_w(\varphi)/\overline{T}_w$. For a uniform peripheral heat flux (b = 0), the wall-temperature difference is given by equation (41). Then, comparing equations (41) and (46) shows that

$$\overline{T}_w = (T_w)_{q=\overline{q}} \tag{48}$$

so that $\overline{T}_w$ corresponds to the wall-temperature difference for uniform circumferential heat flux. Thus, equation (47) represents the variation of T_w around the tube periphery due to the heat flux $q = \overline{q}(1 + b \cos \varphi)$ compared with the value for a uniform circumferential heat flux.

Equation (47) has been plotted in figure 8 as a function of the parameters φ, b, and m. It is seen that there is a significant variation of the dimensionless wall-temperature difference $T_w(\varphi)/\overline{T}_w$ around the tube periphery in the presence of a nonuniform peripheral heat flux. Increased value of the heat-flux parameter b results in increased temperature variation, for a given velocity distribution. In addition, the velocity distribution has a pronounced influence on the variation of wall-temperature difference $T_w(\varphi)/\overline{T}_w$ around the tube for a given value of the parameter b. For a value of the heat-flux parameter b = 1, for example, the ratio $T_w(\varphi)/\overline{T}_w$ at $\varphi = 0^\circ$ is about 3.2 for Poiseuille flow (m = 2) but is 5.0 for slug flow ($m \rightarrow \infty$), an increase of 56 percent. These results indicate that (1) the influence of circumferential heat-flux variation on wall temperatures can be quite significant, with maximum temperature differences exceeding average temperature differences by substantial percentages and (2) the effect of velocity distribution on wall temperatures is also of importance, so that the use of Poiseuille flow results for the estimation of wall-temperature variation is not satisfactory when the fluid under consideration exhibits a high degree of non-Newtonian behavior.

The local Nusselt number is obtained readily from its definition given by equation (20) as

$$\mathrm{Nu}(\varphi) = \frac{1 + b \cos \varphi}{f(m) + \frac{b}{2} \cos \varphi} \tag{49}$$

The Nusselt number is infinite at the point where the wall-temperature difference is zero, which corresponds to values of φ given by

$$\cos \varphi \Big|_{\mathrm{Nu}\to\infty} = -\frac{2f(m)}{b} \tag{50}$$

The values of the function f(m) as a function of the velocity parameter m are shown in figure 9. For a given value of b and m, figure 9 and equation (50) can be used to determine the peripheral point where the Nusselt number becomes infinite.

The extreme dimensionless temperature differences that occur at $\varphi = 0^{\circ}$ and $\varphi = 180^{\circ}$ are shown in figure 10 as functions of the parameters b and m. It is noted that, for a given velocity distribution, the temperature ratio is increased as the parameter b is increased. In addition, it is seen that, for a specified value of b, the temperature ratio likewise is increased as the velocity parameter m is increased. The importance of considering the influence of both the heat-flux distribution and the velocity distribution on the extreme temperature differences is evident, and serious problems could emerge if proper consideration is not given to these combined effects.

The local Nusselt number, equation (49), has been evaluated as a function of φ for several values of the parameters b and m, and the results are plotted in figure 11.

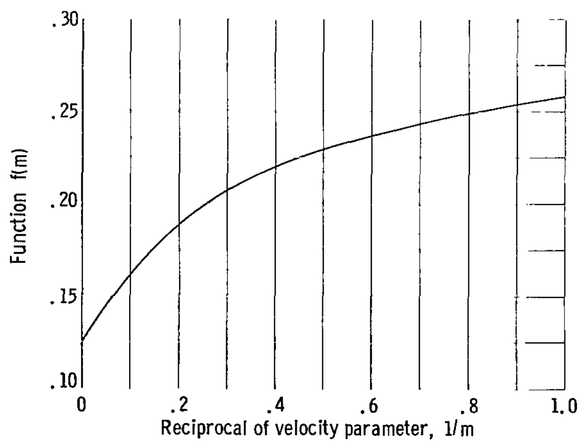


Figure 9. - Function f(m) against reciprocal of velocity parameter.

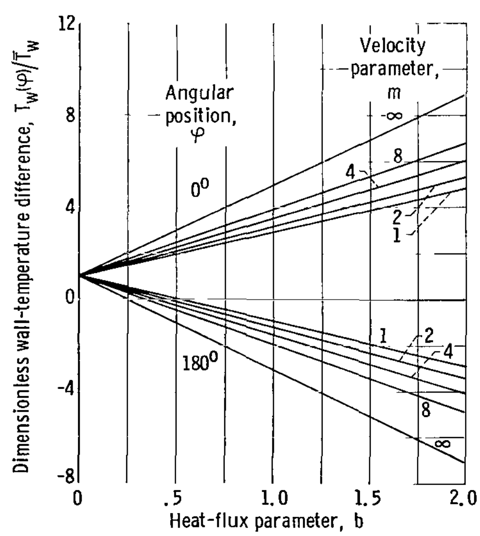


Figure 10. - Maximum wall-temperature differences.

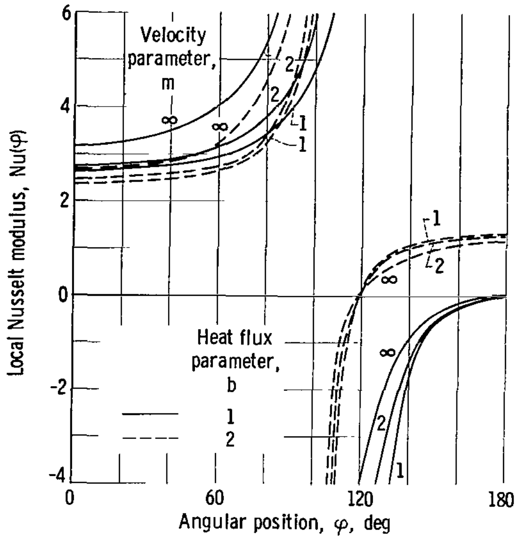


Figure 11. - Local Nusselt modulus variation for prescribed wall heat flux $q(\varphi) = \bar{q}(1 + b \cos \varphi)$.

It is seen that, for a given velocity distribution, the Nusselt modulus, and, hence, the heat-transfer coefficient, varies significantly around the tube periphery for the cosine circumferential heat-flux distribution. For a given value of the heat-flux parameter b, moreover, the velocity distribution markedly effects the heat-transfer coefficient at all peripheral locations. These results point out once more the importance of considering the influence of the velocity profile, or the degree of the non-Newtonian behavior of the fluid, on the heat-transfer coefficient.

CONCLUDING REMARKS

An analysis has been carried out to evaluate the effects of velocity distribution on fully developed laminar heat transfer in a circular tube with a prescribed wall temperature or wall heat flux.

If, in a specific heat-transfer problem, the velocity profile is determined, it may be matched to the profiles as shown in figure 1 (p. 3), and the velocity parameter m determined. For any given value of m, quantities of engineering interest such as wall temperature, wall heat flux, and Nusselt modulus variations around the tube periphery can be calculated.

The influence of the velocity distribution has been demonstrated by evaluating a few illustrative numerical examples, which indicate that the influence of circumferential wall-temperature variation on wall heat fluxes and heat-flux variation on wall temperatures can be significant. In addition, it is found that the degree of non-Newtonian behavior of the flowing fluid, characterized by the value of the velocity parameter m, has an important influence on these quantities, which means that, under these circumstances, the use of Poiseuille flow results ($m = 2$) for the estimation of peripheral wall temperature distributions is not satisfactory.

In the examples illustrated herein, infinities and zeros occur in the Nusselt numbers. The Nusselt numbers become infinite when the bulk mean temperature of the fluid is equal to the appropriate wall temperature. Zero Nusselt numbers indicate a change in the direction of the heat transfer at that angular position of the tube wall. The occurrence of negative, zero, and infinite values of the heat-transfer coefficient about the tube periphery suggests that the Nusselt modulus is of little use when the circumferential wall temperature or heat flux is variable.

Wall-conduction effects will be expected to moderate, to some extent, the wall temperature or wall heat-flux variations (refs. 5 and 6). The results given here represent the extreme cases where the wall heat conduction is negligible compared with the convective heat transfer within the fluid, and the use of the present results should lead to a conservative design of heat-exchange equipment.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 12, 1964.

APPENDIX - SYMBOLS

a	dimensionless parameter, $2\,\frac{m+2}{m}\,\frac{q_w r_o}{k}\,\frac{\theta}{\pi}$
a_n	Fourier coefficients of eq. (1)
b	heat-flux parameter given by eq. (43)
b_n	Fourier coefficients of eq. (1)
c_n	Fourier coefficients of eq. (29)
c_p	heat capacity at constant pressure
$F(\eta, \varphi)$	function of η and φ for variable circumferential heat-flux problem given by eq. (25)
$f(m)$	function of m given by eq. (14)
$G(\eta)$	function of η for variable circumferential heat-flux problem given by eq. (25)
$H_n(\eta)$	radial distribution function for variable circumferential wall-temperature problem given by eq. (6)
k	fluid thermal conductivity
m	velocity profile parameter defined by eq. (4)
$Nu(\varphi)$	local Nusselt number, $q(\varphi)(2r_o)/k[t_w \varphi) - t_b]$
$\overline{Nu}$	circumferentially averaged Nusselt number, $\bar{q}(2r_o)/k(\bar{t}_w - t_b)$
Q	heat-transfer rate per unit length
q	local heat flux
$\bar{q}$	average heat-flux rate per unit heated area
q_w	average heat flux rate at wall over $-\theta \leq \varphi \leq \theta$
r	radial coordinate
r_o	tube radius
T	temperature difference for variable circumferential heat flux, $t - t_b$
$T_w(\varphi)$	wall- to bulk-temperature difference for variable circumferential heat flux, $t_w(\varphi) - t_b$
$\overline{T}_w$	average wall- to bulk-temperature difference for circumferential heat flux, $\bar{t}_w - t_b$
t	fluid temperature
u	velocity
x	axial coordinate
α	thermal diffusivity, $k/\rho c_p$
η	dimensionless coordinate, r/r_o
θ	half angle of heated tube segment
ξ	dummy integration variable

ρ	fluid density
τ	dimensionless temperature difference for variable circumferential wall temperature, $(t - \bar{t}_w)/(\bar{q}r_o/k)$
φ	angular coordinate, deg

Subscripts:

b	fluid bulk condition
c	centerline
d	axisymmetric heating condition
w	wall

Superscript:

$(\bar{\ })$	average value

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